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Optical Booster for Dielectric Laser Accelerators

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Abstract. We present a study of adiabatic tapering of a dielectric laser accelerator and the dynamics of the trapping process. The characteristics of the trapped electrons were studied for different initial conditions. Space-charge effects on the longitudinal motion were considered as well.

Progress during the last decade in wall-plug-to-light efficiency of lasers makes them serious competitors to current microwave-driven accelerators for high energy physics as well as for medical applications. One of the aspects that makes the optical system significantly different, compared to microwave machines, is the *trapping* condition. In the case of a *uniform* acceleration structure operating at the speed of light, the condition for micro-bunch trapping [1] for a given initial velocity ($c\beta_{\rm in}$) is

$$E_{\min} = 2\pi \frac{mc^2}{e\lambda} \sqrt{\frac{1-\beta_{\rm in}}{1+\beta_{\rm in}}}\,. \tag{1}$$

For realistic gradients ($E_0 = 1-10\ GV/m$), the typical value of the normalized longitudinal field $a \equiv eE_0\lambda / mc^2$ is much smaller than unity, as compared with the case of a conventional RF photo-injector, where typically $a \simeq 1$ for $\lambda = 10\ cm$. Consider as an example the case of $\lambda = 1\ \mu m$ and zero initial energy, the required gradient to capture sub-relativistic particles is of the order of 3 TV/m. For comparison, in the microwave regime the trapping gradient is ~ 20MV/m, for which, evidently, the electrons become relativistic within a few wavelengths. Consequently, in the optical regime, there is a need for a tapered structure to maintain local phase synchronicity with the particles.

In the framework of this paper, we present a longitudinal tapering of the structure to maximize the trapping efficiency. We further solve the dynamics of the trapping process with\without longitudinal space-charge effects for both the resonant particle and a distribution of particles. We investigate the effect of the initial phase and energy distribution on the trapped electrons and the latter's characteristics.

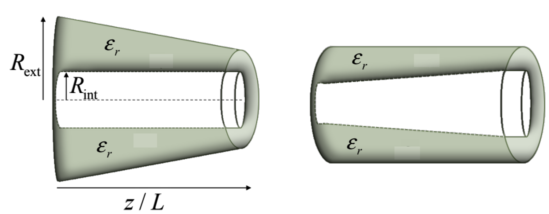

FIGURE 1. Schematic of a tapered booster. (left) internal radius constant, (right) external radius constant.

For the numerical simulations, we adopt a configuration of a dielectric-loaded cylindrical waveguide. Adjusting the relative phase between the wave and the slow electrons can be done by varying the dielectric coefficient ε_r along the z axis, or by changing the radius of the waveguide. For the latter case, Fig. 1 describes two potential types of tapered structures, each keeping either the external or the internal radius uniform. Without significant loss of generality, throughout our analysis we assume that the vacuum tunnel's radius (internal radius) is constant, and only single mode operation is considered. Also, since we are interested not only in the effect of the field on the electrons,

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but also vice-versa, the relation between the interaction impedance and the phase velocity should be determined locally (adiabatic tapering).

ADIABATIC TAPERING

The *first step* is to evaluate the relationship between the *interaction impedance* and the structure's dimensions to the *phase velocity* of the wave which is synchronous with the (resonant) particle. Note that, while our calculation here represents a specific geometry, a similar assessment can be made numerically for a more complex structure. For single-mode operation with an internal radius R_{int}, the external radius R_{ext} is set by solving the dispersion relation for the first accelerating mode, for each β_{ph} along the waveguide. Defining

$$\begin{aligned} T_0(\kappa R_{\text{int}}) &\equiv J_0(\kappa R_{\text{int}}) Y_0(\kappa R_{\text{ext}}) - J_0(\kappa R_{\text{ext}}) Y_0(\kappa R_{\text{int}}) \\ T_1(\kappa R_{\text{int}}) &\equiv J_1(\kappa R_{\text{int}}) Y_0(\kappa R_{\text{ext}}) - J_0(\kappa R_{\text{ext}}) Y_1(\kappa R_{\text{int}}) \end{aligned}, \tag{2}$$

where J_n and Y_n are the Bessel function of the first and second kind, respectively, and $\kappa = \omega\sqrt{\varepsilon_r - \beta_{\text{ph}}^{-2}}/c$ is the transverse wavenumber in the dielectric. Further, introducing the function $I_c(x) \equiv 2I_1(x)/x$, where I_1 is the modified Bessel function of the first kind, we get the dispersion relation

$$\varepsilon_r I_0(\Gamma R_{\text{int}}) T_1(\kappa R_{\text{int}}) - 0.5\kappa R_{\text{int}} I_c(\Gamma R_{\text{int}}) T_0(\kappa R_{\text{int}}) = 0. \tag{3}$$

In (3), $\Gamma = \omega\sqrt{\beta_{\text{ph}}^{-2} - 1}/c$ is the transverse wavenumber in the vacuum. Once the geometry is determined, we can establish the dependence of the interaction impedance (Z_{int}) on the phase velocity and the electron beam radius R_{b}. Figure 2 reveals this relation for sub-relativistic electrons in an accelerating field driven by an IR laser $(\lambda = 1\ \mu m)$. The design is for an initial electron energy of 40 keV, which would correspond to $\beta_{\text{ph}} = 0.373$, and a final energy of 10 MeV ($\gamma_{\text{out}} \simeq 20.6$). Obviously, we tacitly assume adiabatic tapering and no additional modes are excited when varying the geometry.

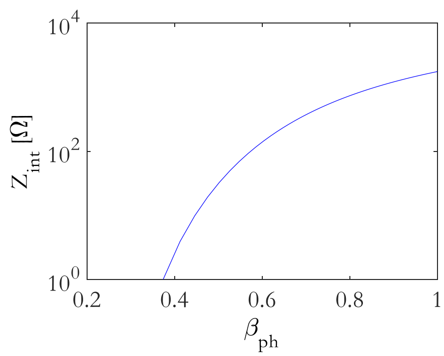

FIGURE 2. Interaction impedance as a function of the phase velocity for $\lambda = 1\ \mu m$ drive laser wavelength, which co-propagates with the electron beam. The latter's radius is $R_{\text{b}} = 0.1R_{\text{int}}$, whereas the cylindrical vacuum tunnel is $R_{\text{int}} = 0.3\lambda$. The structure's length is $L = 0.3\,mm$ and the dielectric loading is made from silicon with $\varepsilon_r = 11.68$.

This longitudinal tapering of the DLA accelerator is, in practice, a variation of the axial wavenumber $k(z) \equiv \omega/[c\beta_{\text{ph}}(z)]$ as a function of the location of the resonant particle $\beta_{\text{ph}}(z) = \beta_r(z)$. Evidently, the local phase velocity needs to be synchronous with the local velocity of the resonant particle. We previously tapered the wavenumber for a constant amplitude [2], and, thus, the *next step* would be to taper the amplitude as well. The longitudinal accelerating field for a traveling-wave accelerator would then be of the form

$$E_z(z,t) = E_0(z)\cos\left[\chi(0) + \omega t - \int_0^z dz' k(z')\right], \tag{4}$$

where $\chi(0)$ is the initial phase.

DYNAMICS IGNORING THE SPACE-CHARGE EFFECT

In order to derive the differential equations of the particle motion, let us assume the existence of an infinitely strong magnetic field, which allows us to ignore the radial dynamics of the beam. Secondly, it is assumed that the radius of the vacuum tunnel (R_{int}) is constant, and only single-mode operation is considered. Thirdly, since we are interested not only in the effect of the field on the electrons, but also vice-versa, it is assumed that the relation between interaction impedance and the phase velocity is known from the previous section, and is determined locally (adiabatic tapering).

With these assumptions in mind, we formulate the interaction dynamics between a wave and an ensemble of N particles. It is assumed that the distribution of the ensemble in one period of the wave is known. The phase of the i-th particle χ_i is relative to the wave, its energy is γ_i, and its radial location is r_i. In a *uniform* structure, given initial conditions (r_i,γ_i,χ_i,a) and assuming no reflected electrons, the dynamics along the interaction space $\zeta \equiv z/\lambda$ is described by

$$\begin{cases} \dfrac{\mathrm{d}a}{\mathrm{d}\zeta} = \alpha\left\langle \mathrm{I}_0\left(\Gamma r_i\right)\exp\left(-j\chi_i\right)\right\rangle \\ \dfrac{\mathrm{d}\gamma_i}{\mathrm{d}\zeta} = -\dfrac{1}{2}\left[a\mathrm{I}_0\left(\Gamma r_i\right)\exp\left(j\chi_i\right)+\text{c.c.}\right] \\ \dfrac{\mathrm{d}\chi_i}{d\zeta} = 2\pi\left[\dfrac{1}{\beta_i}-\dfrac{1}{\beta_{\text{ph}}\left(\zeta\right)}\right] \end{cases}, \quad (5)$$

where $a = eE_0\lambda/mc^2$, $\alpha\left(\beta_{\text{ph}}\right) = eIZ_{\text{int}}\left(\beta_{\text{ph}}\right)/mc^2$ and the current is $I = eN_{\text{el}}c/\lambda$. $\langle\ldots\rangle$ is the average of all the particles $i = 1,2..N$ in the ensemble. In the case of a *tapered* structure, we define $b = a/\sqrt{\alpha}$, which implies that energy $(\langle\gamma_i\rangle + |b|^2/2)$ is conserved. Then, the dynamic is described by

$$\begin{cases} \dfrac{\mathrm{d}b}{\mathrm{d}\zeta} = \sqrt{\alpha}\left\langle \mathrm{I}_0\left(\Gamma r_i\right)\exp\left(-j\chi_i\right)\right\rangle \\ \dfrac{\mathrm{d}\gamma_i}{\mathrm{d}\zeta} = -\dfrac{1}{2}\left[\sqrt{\alpha}b\mathrm{I}_0\left(\Gamma r_i\right)\exp\left(j\chi_i\right)+\text{c.c.}\right] \\ \dfrac{d\chi_i}{d\zeta} = 2\pi\left[\dfrac{1}{\beta_i}-\dfrac{1}{\beta_{\text{ph}}\left(\zeta\right)}\right] \end{cases}. \quad (6)$$

In order to determine how the structure should vary in space, we assume that all the charge concentrates in one micro-particle we shall refer to as the *resonant-particle*, whose local velocity establishes by the synchronization condition, the phase velocity dependence on the longitudinal coordinate. For the sake of keeping a particle in resonance with the wave along the entire structure, the former should retain a constant acceleration phase $\chi_{\text{r}} = \pi$ with the latter, i.e. $d\chi_{\text{r}}/d\zeta = 0$.

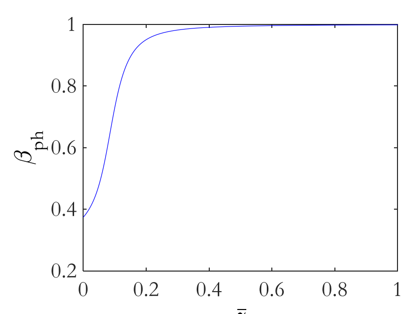

FIGURE 3. The resonant particle phase velocity dependence on the longitudinal coordinate $\bar{z} = z/L$ for the same parameters as in Fig. 2.

Indicated by subscript r, Eqs. (6) for the resonant particle on axis, $r_{\mathrm{r}}=0$, are reduced to $db_{\mathrm{r}}/d\zeta=-\sqrt{\alpha(\beta_{\mathrm{ph}})}$, $d\gamma_{\mathrm{r}}/d\zeta=\sqrt{\alpha(\beta_{\mathrm{ph}})}b_{\mathrm{r}}$. The solution for $\beta_{\mathrm{ph}}(\zeta)$ is shown in Fig. 3; we assume an adiabatic taper, and indeed the major change is over the first 20% of the structure.

Next, we solve the dynamics (Eq. (6)) for 360 particles which are distributed uniformly in energy and phase over the range $[\gamma_{\mathrm{in}}-\delta\gamma,\gamma_{\mathrm{in}}+\delta\gamma]$ and $[\pi-\delta\chi,\pi+\delta\chi]$ respectively. Each particle has an energy $\gamma_i=\gamma_{\mathrm{in}}\pm\delta\gamma_i$, phase $\chi_i=\pi\pm\delta\chi_i$, and radial location $r_i\in[0,R_b]$. Figure 4 describes the phase space dynamics along the travel distance $\bar{z}=z/L$ for $\delta\gamma=10^{-3}\gamma_{\mathrm{in}}$, $\delta\chi=0.05\pi$.

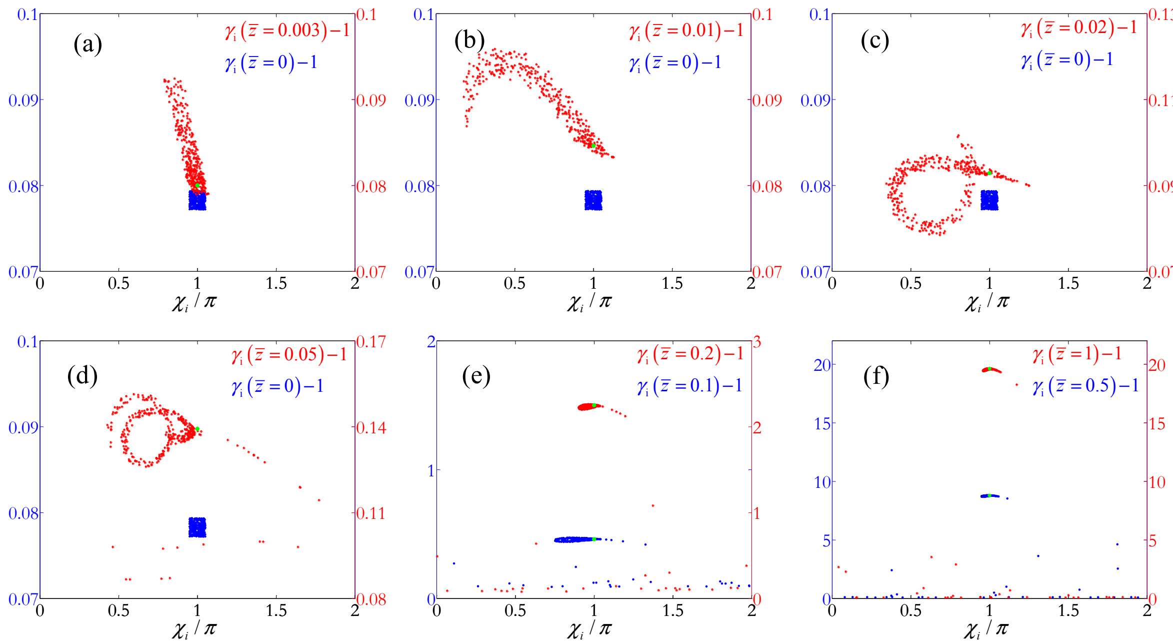

FIGURE 4. Phase space dynamics of 360 electrons with a uniform distribution of $\delta\gamma=10^{-3}\gamma_{\mathrm{in}}$, $\delta\chi=0.05\pi$ along their travel distance $\bar{z}\equiv z/L$. The resonant particle is highlighted in *green*.

In Fig. 4(a) to (d), the propagation through the first 0.3% to 5% of the waveguide is described in red, and for reference, the initial distribution of the phase space is described in blue. As one can see, most of the dynamics occur over the first 20λ of the waveguide (total length is $L=300\lambda$). The first change is in the energy spread (Fig. 4(a)), and then also in the phase (Fig. 4(b)). After 10% of the waveguide, there is a clear distinction to the two separate groups, the accelerated and the decelerated bunches, as shown in blue in Fig. 4(e). From that point and on, the accelerated bunch continues to gain energy, whereas the decelerated electrons remain behind and can be found in all phases from 0 to 2π. At the exit of the waveguide, the accelerated bunch reaches the energy that the structure was designed for (10 MeV), as shown in Fig. 4(f). In all parts of the figure, the resonant particle $(\gamma_{\mathrm{r}},\ \chi_{\mathrm{r}}=\pi)$ is highlighted in green.

With this dynamic in mind, the question is how many electrons are trapped, and what is their final energy spread. The trapping criterion was set to be a final electron kinetic energy that is higher than 80% of the resonant particle energy for which the structure was designed, i.e. $|\gamma_{\mathrm{out,i}}-1|>0.8(\gamma_{\mathrm{out}}-1)$. To answer this question, we consider the initial uniform phase distribution of $\delta\chi=0.05\pi$, and a range of initial energy spread $\Delta\gamma_{\mathrm{in}}$. Figure 5 shows the trapped fraction of the electrons (in blue) and final energy spread $\Delta\gamma_{\mathrm{out}}$ (in red) for each initial condition. As the initial energy spread is smaller, more electrons are trapped, and their final energy spread is smaller.

At the other extreme, when the initial energy spread is higher, less particles are trapped, and the variation in the latter's final energy spread is higher. Nevertheless, the final energy spread of the trapped particles does not change significantly!

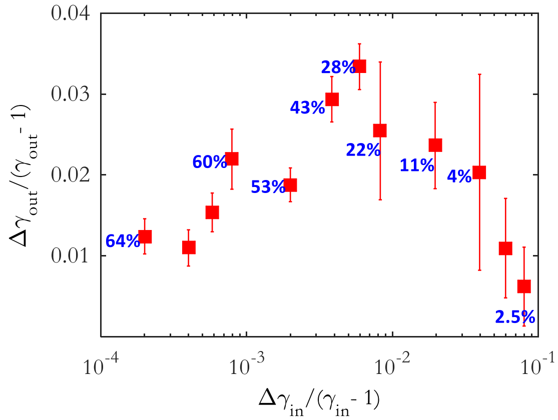

FIGURE 5. Fraction of trapped electrons and their final energy spread, for different values of initial energy spread.

LONGITUDINAL SPACE CHARGE

For an assessment of the longitudinal space-charge effect, the energy equation takes the form (Chapter 6 in [1])

$$\frac{d}{d\zeta}\gamma_i = -\frac{1}{2}\left[\left(\sqrt{\alpha}\, b I_0(\Gamma r_i) - j\frac{\bar{I}\cdot\lambda_L}{R_{\text{int}}}\overline{\xi_{sc}}\left\langle e^{-j\chi_{i'}}\right\rangle_{i'}\right)\exp(j\chi_i) + \text{c.c.}\right], \quad (7)$$

where $\bar{I} = \eta_0 Ie/mc^2$ is the normalized current, $\eta_0 = 377\Omega$ is the wave impedance, the normalized space charge (SC) coefficient for cylindrical waveguide is $\overline{\xi_{sc}} = \xi_{sc}(R_{\text{int}}/R_b)^2(\pi\omega R_{\text{int}}/c)^{-1}$, the SC coefficient is $\xi_{sc} = 1 - \sum_{s=1}^{N}\left[J_1(p_s R_b/R_{\text{int}})/\Delta_s J_1(p_s)\right]^2$, $\Delta_s = \sqrt{p_s^2 - (\omega R_{\text{int}}/c)^2}$, and p_s are the zeros of the Bessel function of the first kind. When the SC effect is included, less electrons were accelerated (63%), as compared with the case where the SC is ignored (73%). However, the difference diminishes for larger radii of the e-beam.

SUMMARY

We presented a complete formulation of the dynamics of sub-relativistic electrons in an adiabatically tapered laser-based acceleration structure, addressing the beam-loading effect. Next, we investigated the trapping process of both the resonant particle, as well as a distribution of particles, and the energy spread of the trapped electrons. Finally, when accounting for longitudinal space-charge forces, the percentage of trapped electrons decreases.

ACKNOWLEDGMENT

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